

Module 2 DC Circuit

Lesson

4

Loop Analysis of resistive circuit in the context of dc voltages and currents

Objectives

- Meaning of circuit analysis; distinguish between the terms mesh and loop.
- To provide more general and powerful circuit analysis tool based on Kirchhoff's voltage law (KVL) only.

L.4.1 Introduction

The Series-parallel reduction technique that we learned in lesson-3 for analyzing DC circuits simplifies every step logically from the preceding step and leads on logically to the next step. Unfortunately, if the circuit is complicated, this method (the simplify and reconstruct) becomes mathematically laborious, time consuming and likely to produce mistake in calculations. In fact, to elevate these difficulties, some methods are available which do not require much thought at all and we need only to follow a well-defined faithful procedure. One most popular technique will be discussed in this lesson is known as 'mesh or loop' analysis method that based on the fundamental principles of circuits laws, namely, Ohm's law and Kirchhoff's voltage law. Some simple circuit problems will be analyzed by hand calculation to understand the procedure that involve in mesh or loop current analysis.

L.4.1.1 Meaning of circuit analysis

The method by which one can determine a variable (either a voltage or a current) of a circuit is called analysis. Basic difference between 'mesh' and 'loop' is discussed in lesson-3 with an example. A 'mesh' is any closed path in a given circuit that does not have any element (or branch) inside it. A mesh has the properties that (i) every node in the closed path is exactly formed with two branches (ii) no other branches are enclosed by the closed path. Meshes can be thought of a resembling window partitions. On the other hand, 'loop' is also a closed path but inside the closed path there may be one or more than one branches or elements.

L.4.2 Solution of Electric Circuit Based on Mesh (Loop) Current Method

Let us consider a simple dc network as shown in Figure 4.1 to find the currents through different branches using Mesh (Loop) current method.

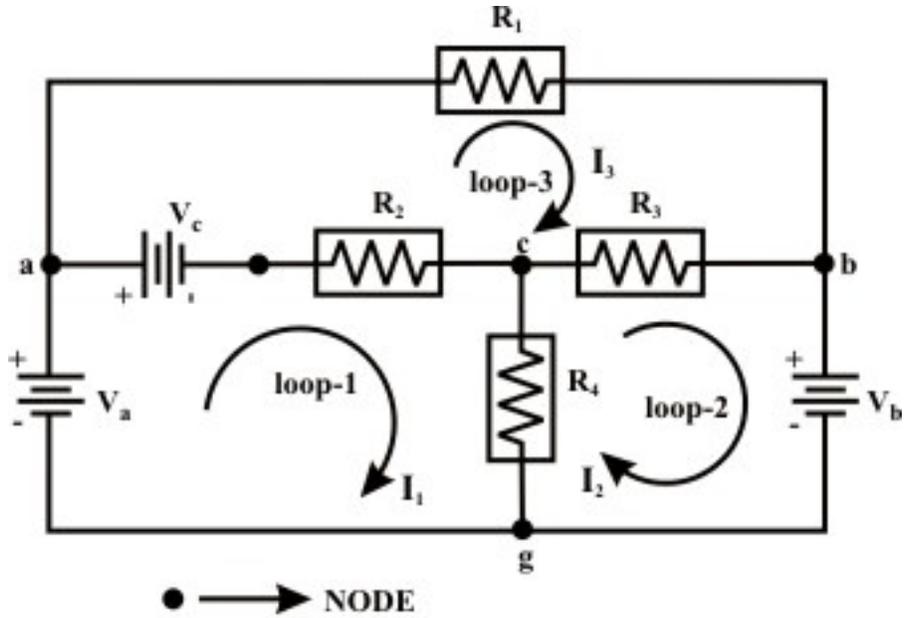


Figure 4.1

Applying KVL around mesh (loop)-1:(note in mesh-1, I_1 is known as local current and other mesh currents I_2 & I_3 are known as foreign currents.)

$$V_a - V_c - (I_1 - I_3)R_2 - (I_1 - I_2)R_4 = 0$$

$$V_a - V_c = (R_2 + R_4)I_1 - R_4I_2 - R_2I_3 = R_{11}I_1 - R_{12}I_2 - R_{13}I_3 \quad (4.1)$$

Applying KVL around mesh (loop)-2:(similarly in mesh-2, I_2 is local current and I_1 & I_3 are known as foreign currents)

$$-V_b - (I_2 - I_3)R_3 - (I_2 - I_1)R_4 = 0$$

$$-V_b = -R_4I_1 + (R_3 + R_4)I_2 - R_3I_3 = -R_{21}I_1 + R_{22}I_2 - R_{23}I_3 \quad (4.2)$$

Applying KVL around mesh (loop)-3:

$$V_c - I_3R_1 - (I_3 - I_2)R_3 - (I_3 - I_1)R_2 = 0$$

$$V_c = -R_2I_1 - R_3I_2 + (R_1 + R_2 + R_3)I_3 = -R_{31}I_1 - R_{32}I_2 + R_{33}I_3 \quad (4.3)$$

** In general, we can write for i^{th} mesh (for $i = 1, 2, \dots, N$)

$$\sum V_{ii} = -R_{i1}I_1 - R_{i2}I_2 \dots \dots + R_{ii}I_i - R_{i,i+1}I_{i+1} - \dots - R_{iN}I_N$$

$\sum V_{ii} \rightarrow$ simply means to take the algebraic sum of all voltage sources around the i^{th} mesh.

$R_{ii} \rightarrow$ means the total self resistance around the i^{th} mesh.

$R_{ij} \rightarrow$ means the mutual resistance between the and j^{th} meshes.

Note: Generally, $R_{ij} = R_{ji}$ (true only for linear bilateral circuits)

$I_i \rightarrow$ the unknown mesh currents for the network.

Summarize:

Step-1: Draw the circuit on a flat surface with no conductor crossovers.

Step-2: Label the mesh currents (I_i) carefully in a clockwise direction.

Step-3: Write the mesh equations by inspecting the circuit (No. of independent mesh (loop) equations=no. of branches (b) - no. of principle nodes (n) + 1).

Note:

To analysis, a resistive network containing voltage and current sources using 'mesh' equations method the following steps are essential to note:

- If possible, convert current source to voltage source.
- Otherwise, define **the voltage** across the current source and write the mesh equations as if these source voltages were known. Augment the set of equations with one equation for each current source expressing a known mesh current or difference between two mesh currents.
- Mesh analysis is valid only for circuits that can be drawn in a two-dimensional plane in such a way that no element crosses over another.

Example-L-4.1: Find the **current** through 'ab-branch' (I_{ab}) and **voltage** (V_{cg}) across the current source using Mesh-current method.

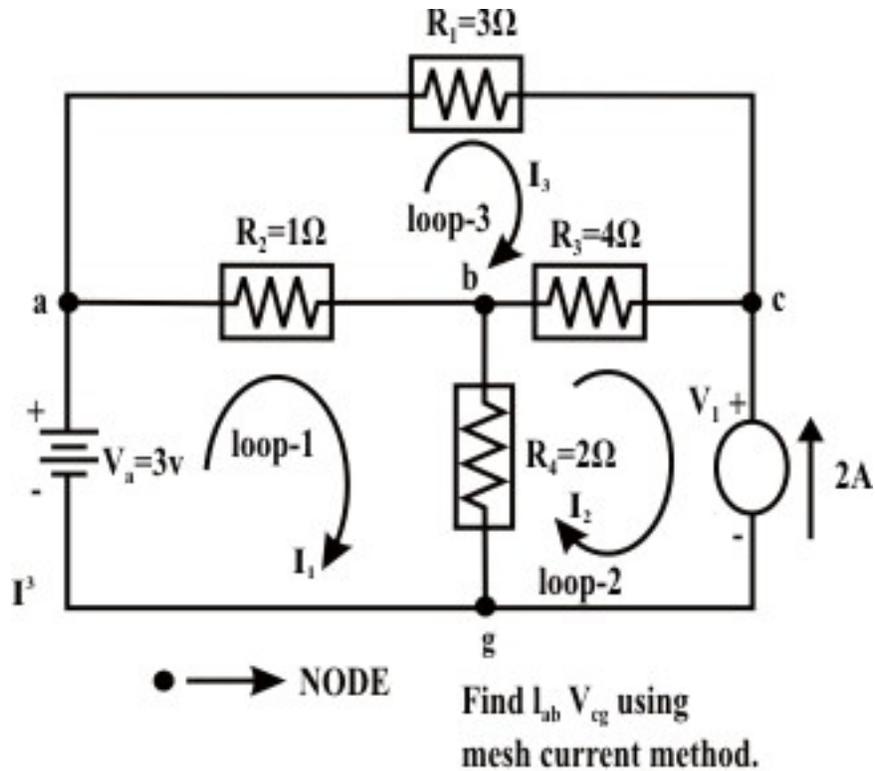


Figure 4.2

Solution: Assume voltage across the current source is v_1 ('c' is higher potential than 'g' (ground potential and assumed as zero potential) and note $I_2 = -2A$ (since assigned current direction (I_2) is opposite to the source current)

Loop - 1: (Applying KVL)

$$V_a - (I_1 - I_3)R_2 - (I_1 - I_2)R_4 = 0 \Rightarrow 3 = 3I_1 - 2I_2 - I_3$$

$$3I_1 - I_3 = -1 \tag{4.4}$$

Loop - 2: (Applying KVL)

Let us assume the voltage across the current source is v_1 and its top end is assigned with a positive sign.

$$-v_1 - (I_2 - I_1)R_4 - (I_2 - I_3)R_3 = 0 \Rightarrow -v_1 = -2I_1 + 6I_2 - 4I_3$$

$$2I_1 + 12 + 4I_3 = v_1 \quad (\text{note: } I_2 = -2A) \tag{4.5}$$

Loop - 3: (Applying KVL)

$$-I_3 R_1 - (I_3 - I_2)R_3 - (I_3 - I_1)R_2 = 0 \Rightarrow -I_1 - 4I_2 + 8I_3 = 0$$

$$I_1 - 8I_3 = 8 \quad (\text{Note, } I_2 = -2A) \tag{4.6}$$

Solving equations (4.4) and (4.6), we get $I_1 = -\frac{48}{69} = -0.6956A$ and

$$I_3 = -\frac{25}{23} = -1.0869A, \quad I_{ab} = I_1 - I_3 = 0.39A, \quad I_{bc} = I_2 - I_3 = -0.913A \quad \text{and}$$

$$I_{bg} = I_1 - I_2 = 1.304A$$

- ve sign of current means that the current flows in reverse direction (in our case, the current flows through 4Ω resistor from 'c' to 'b' point). From equation (4.5), one can get $v_1 = 6.27$ volt.

Another way: $-v_1 + v_{bg} + v_{bc} = 0 \Rightarrow v_1 = v_{cg} = 6.27$ volt.

Example-L-4.2 For the circuit shown Figure 4.3 (a) find V_x using the mesh current method.

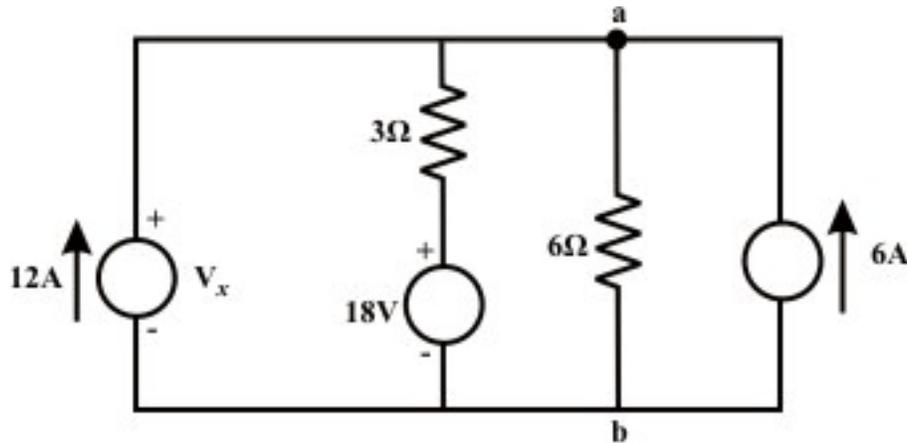


Fig. 4.3(a)

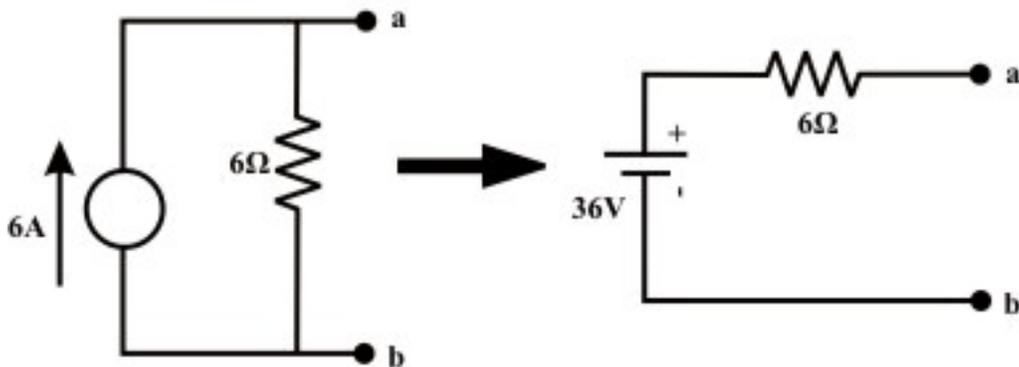


Fig. 4.3(b)

Solution: One can easily convert the extreme right current source (6 A) into a voltage source. Note that the current source magnitude is 6 A and its internal resistance is 6Ω . The given circuit is redrawn and shown in Figure 4.3 (c)

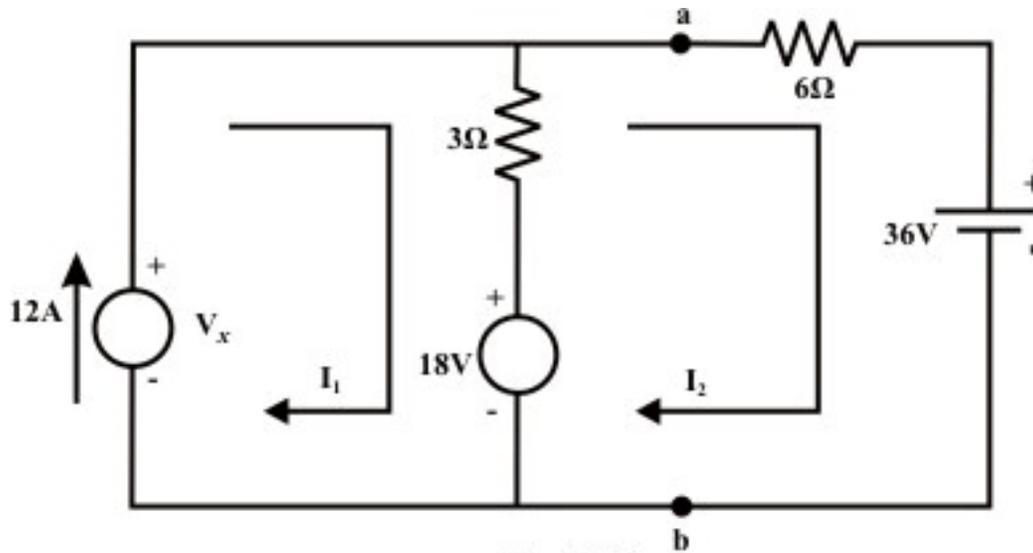


Fig. 4.3(c)

Loop-1: (Write KVL, note $I_1 = 12 A$)

$$V_x - (I_1 - I_2) \times 3 - 18 = 0 \Rightarrow V_x + 3I_2 = 54 \quad (4.7)$$

Loop-2: (write KVL)

$$18 - (I_2 - I_1) \times 3 - I_2 \times 6 - 36 = 0 \Rightarrow 9I_2 = 18 \Rightarrow I_2 = 2 A$$

Using the value of $I_2 = 2 A$ in equation (4.7), we get $V_x = 48 \text{ volt}$.

Example-L-4.3 Find v_R for the circuit shown in figure 4.4 using ‘mesh current method’. Calculate the power absorbed or delivered by the sources and all the elements.

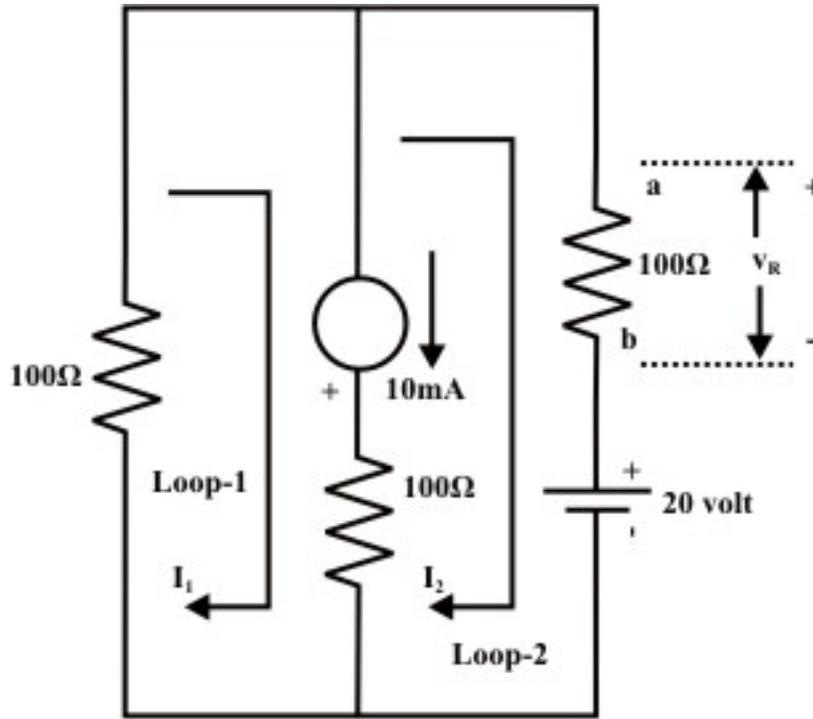


Figure 4.4

Solution: Assume the voltage across the current source is ‘ v ’ and the bottom end of current source is marked as positive sign.

For loop No. 1: (KVL equation)

$$v - (I_1 - I_2) \times 100 - I_1 \times 100 = 0 \Rightarrow v - 200I_1 + 100I_2 = 0 \quad (4.8)$$

It may be noted that from the figure that the current flowing through the 100Ω resistor (in the middle branch) is $10mA$. More specifically, one can write the following expression

$$I_1 - I_2 = 10 \times 10^{-3} \quad (4.9)$$

For loop No. 2: (KVL equation)

$$-20 - (I_2 - I_1) \times 100 - v - I_2 \times 100 = 0 \Rightarrow v + 200I_2 - 100I_1 = -20 \quad (4.10)$$

Solving equations (4.8)–(4.10), one can obtain the loop currents as $I_1 = -0.095 = -95mA$ (-ve sign indicates that the assigned loop current direction is not correct or in other words loop current (I_1) direction is anticlockwise.) and $I_2 = -0.105 = -105mA$ (note, loop current (I_2) direction is anticlockwise). Now the voltage across the 100Ω resistor (extreme right branch) is given by $v_R = I_2 \times 100 = -0.105 \times 100 = -10.5\text{ volt}$. This indicates that the resistor terminal (b) adjacent to the voltage source is more positive than the other end of the resistor terminal

(a). From equation (4.8) $v = -8.5 \text{ volt}$ and this implies that the 'top' end of the current source is more positive than the bottom 'end'.

Power delivered by the voltage source = $20 \times 0.105 = 2.1 \text{ W}$ (note that the current is leaving the positive terminal of the voltage source). On the other hand, the power received or absorbed by the current source = $8.5 \times 0.01 = 0.085 \text{ W}$ (since current entering to the positive terminal (top terminal) of the current source). Power absorbed by the all resistance is given

$$= (0.105)^2 \times 100 + (0.095)^2 \times 100 + (10 \times 10^{-3})^2 \times 100 = 2.015 \text{ W} .$$

Further one can note that the power delivered ($P_d = 2.1 \text{ W}$) = power absorbed ($P_{ab} = 0.085 + 2.015 = 2.1 \text{ W}$) = 2.1 W

L.4.3 Test Your Understanding

[Marks:50]

T.4.1 To write the Kirchhoff's voltage law equation for a loop, we proceed clockwise around the loop, considering voltage rises into the loop equation as ----- terms and voltage drops as ----- terms. [2]

T.4.2 When writing the Kirchhoff's voltage law equation for a loop, how do we handle the situation when an ideal current source is present around the loop? [2]

T.4.3 When a loop current emerges with a positive value from mathematical solution of the system of equations, what does it mean? What does it mean when a loop current emerges with a negative value? [2]

T.4.4 In mesh current method, the current flowing through a resistor can be computed with the knowledge of ----- loop current and ----- loop current. [2]

T.4.5 Find the current through 6Ω resistor for the circuit Figure 4.5 using 'mesh current' method and hence calculate the voltage across the current source. [10]

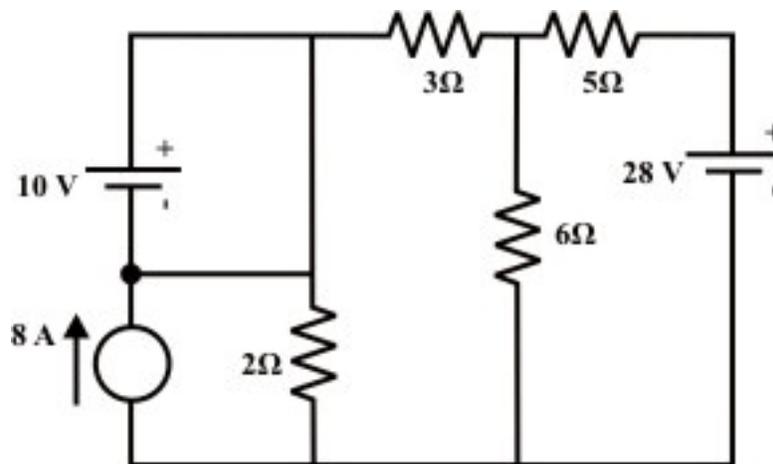


Figure 4.5

(Answer: 3.18 A ; 13.22 V)

T.4.6 For the circuit shown in Figure 4.6, find the current through I_{AB} , I_{AC} , I_{CD} and I_{EF} using ‘mesh current’ method. [12]

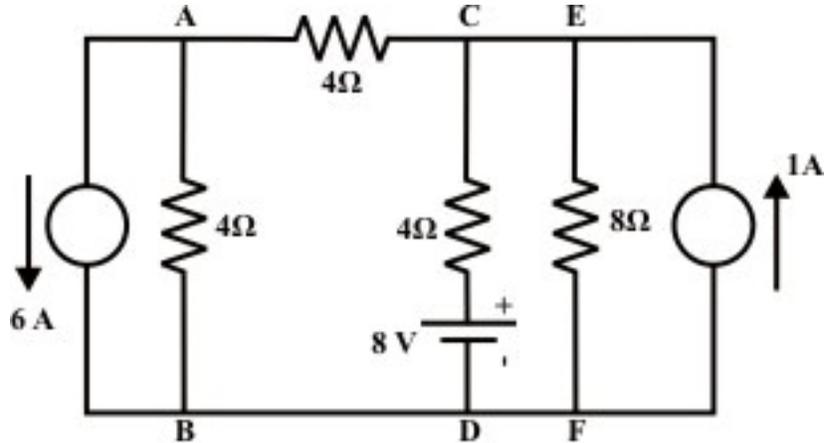


Figure 4.6

(Answer: $I_{AB} = -3\text{ A}$; $I_{AC} = -3\text{ A}$; $I_{CD} = -2\text{ A}$ and $I_{EF} = 0\text{ A}$.)

T.4.7 Find the current flowing through the $R_L = 1\text{ k}\Omega$ resistor for the circuit shown in Figure 4.7 using ‘mesh current’ method. What is the power delivered or absorbed by the independent current source? [10]

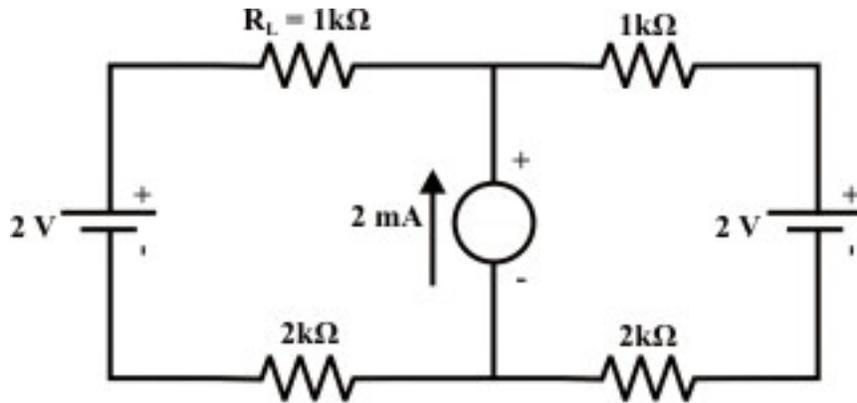


Figure 4.7

(Answer: 1 mA ; 10 mW)

T.4.8 Using ‘mesh current’ method, find the current flowing through 2Ω resistor for the circuit shown in Figure 4.8 and hence compute the power consumed by the same 2Ω resistor. [10]

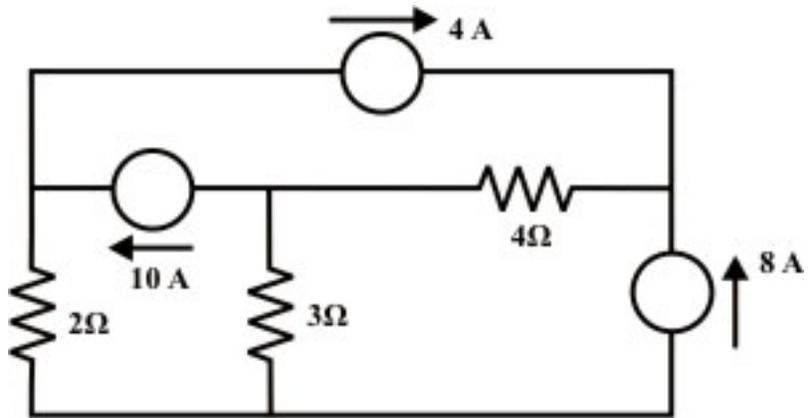


Figure 4.8

(Answer: 6 A; 72W)
